The quantity $y_{int} \leqslant 1.5 \exp(-1/(\varkappa\sqrt{2\delta}))/\sqrt{\delta}$. For small values of z_K the volt–ampere characteristics depend weakly on the behavior of the solution of the problem (1.12)-(1.16) in the interval $(z_K, -z_K)$. An analogous conclusion, based on the results of numerical calculations, was drawn in [6].

In real lasers the neutral gas is in motion. As a result of this, there is a convective removal of the Joule heat and a viscous boundary layer is formed in the regions near the electrodes. The density of the gas can no longer be considered constant. This means that the calculation of the parameters of the flow in the electric field must be carried out on the basis of a simultaneous solution of the gasdynamics equations and the system (1.1) (with the pressure p replaced by the density $\rho(x)$).

This study may be regarded as one of the steps aimed at the investigation of the properties of system (1.1) with variable density and the calculation of the electrical discharge in real electroionization lasers.

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A PARAXIAL MODEL FOR THE GROWTH OF AN EXTERNALLY MAINTAINED DISCHARGE INFLUENCED BY ITS OWN MAGNETIC FIELD

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A high-power electrical-ionization laser system employing compressed gas involves a high discharge power and large geometrical dimensions. This increases the magnetic field of the spatial discharge, and this begins to influence the motion of the electrons responsible for the ionization. When the Larmor radius for the electrons becomes comparable with the transverse dimensions of the discharge, the distribution of the ionization losses and of the electron density will be substantially inhomogeneous [1].

Here we consider an approximate model for a gas discharge initiated by a high-power relativistic electron beam. An analytic expression for the spatial distribution of the energy absorbed in the discharge is derived for the steady-state case.

A two-dimensional problem can be formulated (Fig. 1) for a typical geometry of the spatial discharge in a laser in which the longitudinal dimension is much larger than the transverse dimension d, $l \ll l_0$ (d, distance between electrodes; l, width of the discharge, which is determined by the width of the beam; and l_0, length of the discharge). A relativistic electron beam with zero velocity spread is injected along the z axis from the cathode, with electron energy U_b and current density j_b. A potential difference U_0 is applied to the electrodes and there is a gas at pressure p_0 (atm) in the space between them.

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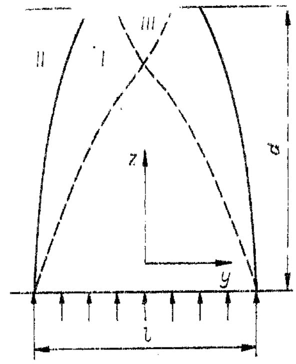

Fig. 1

In a high-power pulse laser, an electron concentration of $\sim 10^{12}$-10^{13} cm^{-3} is attained in a time much less than the characteristic time τ_d for change in the discharge current. The beam charge neutralization and the establishment of a cathode layer require times of several nanoseconds for establishment at such an electron concentration. Further beam injection results in a quasineutral gas-discharge plasma, and the conduction currents are substantially larger than the displacement ones. The characteristic dimensions of the cathode layer are $\sim 10^{-3}$-10^{-4} cm, while the cathode potential drop is only a small part of the potential difference applied to the discharge gap [2], so we can neglect the effects of the cathode layer on the motion of the beam electrons and the discharge dynamics. At the start, there are discharge electrons in the gap, which are produced by some previous ionization and have a density $\sim 10^{12}$-10^{13} cm^{-3}, and therefore the charge neutralization for the beam and the establishment of the cathode layer are not further considered.

The characteristic times for change in the beam current are $\tau_b \gg d/c$, where c is the velocity of light, so the injection of the relativistic beam may be taken as quasistationary. The assumptions of quasineutrality and quasistationary behavior are not applicable for the excitation of discharge-plasma oscillations by the beam, but it can be shown that the electric fields arising in such oscillations are small for the currents usually employed and do not influence the ionization of the relatively dense gas ($p_0 \gtrsim 0.1$ atm) [3].

If τ_b and τ_d are much greater than the skin time* (τ_b, $\tau_d \gg 4\pi l^2\sigma/c^2$, where σ is the gas conductivity), then the induced electric field is small by comparison with the field U_0/d applied to the gap. Then the field may be written as $\mathbf{E} = -\nabla\varphi$.

In what follows we do not consider the scattering of the beam electrons by the gas molecules, which enables us to use the hydrodynamic equations to describe the motion of the beam with a zero pressure tensor. We know of no numerical calculations for self-consistent models for an externally maintained discharge that employ a kinetic equation to describe the motion of the beam electrons. The external electromagnetic field simulates the field from the spatial discharge and the beam, and calculations on the ionization loss for the beam electrons have been performed on this basis [5-7] by Monte Carlo methods. If the energy acquired by a beam electron in the electrode gap is much less than U_b, we can also neglect the effects of the electric field on the motion of the beam electrons. In relation to the above assumptions we must emphasize that we are examining a fairly crude model for a gas discharge that enables one to elucidate the effects of the magnetic field of the discharge on the spatial homogeneity of the absorption of the electrical energy in the discharge gap in a fairly simple fashion.

Within the framework of these approximations, we have the following system of equations for the externally maintained gas discharge:

$$\operatorname{rot} \mathbf{h} = -\frac{4\pi}{c} e\,(\mu n_d \nabla\varphi + n_b \mathbf{v}),$$

$$\frac{\partial n_d}{\partial t} = \frac{1}{\tau_{ib}} n_b - \beta_d n_d^2 + \frac{1}{\tau_{id}} n_d,$$

$$\operatorname{div} n_b \mathbf{v} = 0, \quad (\mathbf{v}\nabla)\,\gamma\mathbf{v} = -\frac{e}{mc}[\mathbf{v}, \mathbf{h}],$$

*The opposite case of beam injection with a given current distribution has been considered [4].

where $\mathbf{h}$ is the magnetic field strength; n_d, β_d, μ, density, recombination coefficient, and mobility for the discharge electrons; n_b and $\mathbf{v}$, density and velocity of the beam electrons; $\gamma = 1 + U_b$; τ_{ib}, ionization time for a beam electron, with τ_{id} the same for a discharge electron; and e and m, electronic charge and mass. We neglect collisional ionization ($\tau_d/\tau_{id} \ll 1$) and the $\mathbf{E}$ dependence of μ, β_d to write the dimensionless equations

$$\text{rot}\,\mathbf{H} = -N_d\nabla\Phi - \eta N_b\mathbf{V}; \quad (1)$$

$$\frac{\partial N_d}{\partial\tau} = N_b - N_d^2; \quad (2)$$

$$\text{div}\,N_b\mathbf{V} = 0; \quad (3)$$

$$(\mathbf{V}\nabla)\mathbf{V} = -2\varkappa[\mathbf{V}\mathbf{H}], \quad (4)$$

where $\mathbf{V} = \mathbf{v}/\beta c$; $N_d = n_d/n_{d0}$; $N_b = n_b e\beta c/j_b$; $h_0 = (4\pi/c)en_{d0}\mu U_0$; $\mathbf{H} = \mathbf{h}/h_0$; $\Phi = \varphi/U_0$; $\mathbf{r}_1 = \mathbf{r}/d$; $\tau = t/\tau_d$; $\eta = j_b d/en_{d0}\mu U_0$; $\varkappa = eh_0 d/2mc^2\beta\gamma$; $\beta = (\sqrt{\gamma^2 - 1})/\gamma$; in eliminating the dimensions it has been assumed that the beam current density at z = 0 is described by $j_b(t) = j_b j(t)$, where j_b is the characteristic beam current, while n_{d0} and τ_d are chosen from the condition

$$n_{d0} = \sqrt{j_b/\tau_{ib}\beta_d e\beta c} \text{ and } \tau_d = 1/\beta_d n_d.$$

Then the following are the dimensionless boundary and initial conditions for (1)-(4):

$$\Phi|_{z_1=0} = 0, \quad \Phi|_{z_1=1} = 1; \quad (5)$$

$$\mathbf{V}|_{z_1=0} = (0, 0, 1); \quad (6)$$

$$N_b|_{z_1=0} = j(\tau)\,\theta\left(y_1 + \frac{l}{2d}\right)\theta\left(-y_1 + \frac{l}{2d}\right); \quad (7)$$

$$N_d|_{\tau=0} = \varepsilon \ll 1, \quad (8)$$

where $\theta(y)$ is a Heaviside step function.

The solution to (1)-(4) is dependent on parameters $\varkappa$, η, l/d and the form of the function $j(\tau)$. In our numerical calculations we used $j(\tau) = 1 - \exp(-\tau/\tau_p)$. We now examine (1)-(4) with (5)-(8) in the paraxial approximation [8], where it is supposed that the beam density varies little along the direction of the beam motion at distances of the order of the transverse compression of the beam, which means that the transverse velocity component V_y is small and also that the transverse fields are weak [9]. This approximation also enables us to use the equations of cold hydrodynamics (3) and (4) with a given density profile for the beam current as in (7). Then we write the longitudinal velocity of the beam as $V_z = 1 - 1/2V_y^2$, where

$$\frac{1}{2}V_y^2 \ll 1, \quad (9)$$

and neglect terms of order $1/2V_y^2$ to reduce (1)-(4) to two equations for $N_d(\mathbf{r}_1, \tau)$ and $I_1(\tau)$:

$$\frac{\partial N_d}{\partial\tau} = j(\tau)\frac{1}{1 - \varkappa(\eta j(\tau) + I_1(\tau))\,z_1^2} - N_p^2; \quad (10)$$

$$\int_0^1 \frac{I_1(\tau)}{N_d}\,\frac{dz_1}{1 - \varkappa(\eta j(\tau) + I_1(\tau))\,z_1^2} = 1, \quad (11)$$

where $I_1 = Id/en_{d0}\mu U_0 l l_0$ and I is the discharge current. We have for the longitudinal component of the electric field that

$$\frac{\partial\Phi}{\partial z_1} = \frac{I_1}{N_d}\,\frac{1}{1 - \varkappa(\eta j(\tau) + I_1(\tau))z_1^2}. \quad (12)$$

Equation (10) describes the discharge density in region I (Fig. 1), which is occupied by the beam at time τ; in region II there is recombinational decay of the gas-discharge plasma. In region I, the densities of the beam and discharge electrons are independent of y_1, while at the boundary with region II, which is described by

$$y_g(z_1, \tau) = \pm\frac{l}{2d}\left(1 - \varkappa(\eta j(\tau) + I_1(\tau))\,z_1^2\right), \quad (13)$$

the normal component of the electric field is zero.

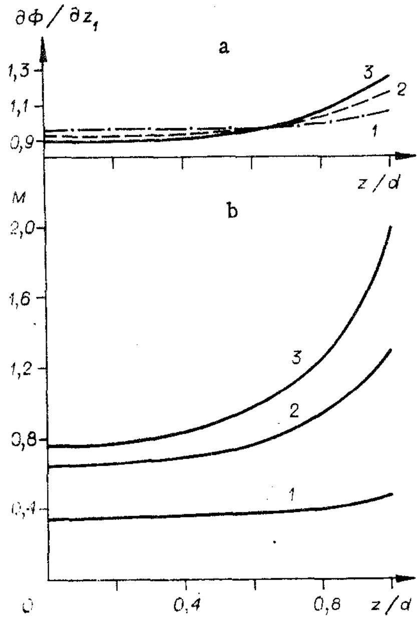

Fig. 2

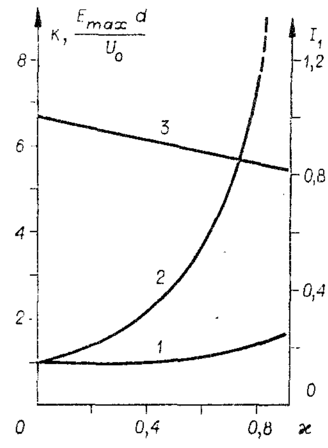

Fig. 3

We see from (13) that (10) and (11) describe the compression of the beam and the discharge. If $\varkappa$ is larger than some value $\varkappa_*$, there is some instant τ_* when with $y_g(z_1 = 1, \tau_*) = 0$ the beam is focused at the anode. On further injection, the contraction in the discharge moves towards the cathode (broken lines in Fig. 1) and in principle several foci may form in the beam in the gap. However, it is impossible to use (10) and (11) for times $\geq\tau_*$ because the discharge-electron density in region III (Fig. 1) is dependent on y_1. In the real situation, the minimum beam width $y_{g\,min}$ is always finite, which is due to the spread in the beam velocities at z = 0 and to diffusion of the beam electrons on account of scattering at the gas molecules. Therefore, this model is not applicable for the focusing region for $\varkappa \sim \varkappa_*$. Remaining within the framework of our model, $y_{g\,min}$ may be estimated by incorporating the terms $\sim V_u^2/2$, and then $y_{g\,min} \gtrsim (l/2d)^3$. System (10) and (11) may be solved numerically for times less than τ_*. Figure 2 shows results for $\varkappa = 0.5$, $\eta = 0.1$ and $\tau_p = 0.3$, viz., the dependence on the distance to the anode for the electric field strength (a) and the discharge power (b) for various instants ($1 - t = 0.6\,\tau_d$, $2 - t = 1.2\,\tau_d$, $3 - t = 1.8\,\tau_d$), where M is the dimensionless discharge power ($M = N_d(\partial\Phi/\partial z_1)^2$), while $\partial\Phi/\partial z_1$ is determined from (12). The inhomogeneity in the discharge power is $K = M_{max}/M_{min}$, which is much greater than the inhomogeneity in the electric field, and the assumption that β_d and μ are independent of E/p_0 is partially justified even for discharges highly inhomogeneous in power. Figure 2 shows that $K \sim 2$ even for $\varkappa = 0.5$ and $t = 1.2\,\tau_d$, i.e., we have a highly inhomogeneous contribution to the energy in the gas.

In the stationary case, where $j(\tau) = 1$ and $\partial N_d/\partial\tau = 0$, the solution to (10) and (11) can be obtained analytically:

$$N_d = \left(\frac{1}{1 - \varkappa(\eta + I_1)z_1^2}\right)^{1/2},$$

where the discharge current I_1 is found from

$$\frac{I_1}{\sqrt{\varkappa(\eta + I_1)}} \arcsin\sqrt{\varkappa(\eta + I_1)} = 1. \tag{14}$$

From (13) and (14) we obtain a value for $\varkappa_*$ that can serve as an estimate for the critical parameters of a real discharge:

$$\varkappa_* = \pi/(2 + \pi\eta). \tag{15}$$

We have the degree of spatial inhomogeneity in the discharge power as

$$K = 1/(1 - \varkappa(\eta + I_1))^{3/2}.$$

Figure 3 shows the maximum overvoltage $E_{max}d/U_0$, K, and the total discharge current I_1 as functions of $\varkappa$ for stationary case for $\eta = 0.1$ (lines 1-3, respectively).

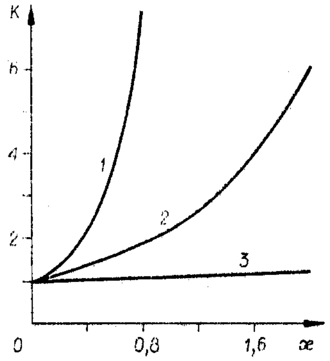

Fig. 4

One of the ways of improving the homogeneity of the discharge is to set up an external magnetic field that compensates the effects of the magnetic field from the spatial discharge [5, 10]. If an external homogeneous magnetic field h_1 is applied to the system (Fig. 1) along the z axis, then system (1)-(4) with condition (9) is again reduced to two equations:

$$\frac{\partial N_d}{\partial \tau} = j(\tau)\frac{1}{1-\varkappa(\eta j(\tau)+I_1(\tau))\frac{\sin^2 \varkappa_1 z_1}{\varkappa_1^2}} - N_d^2,$$
$$\int_0^1 \frac{I_1(\tau)}{N_d}\frac{dz_1}{1-\varkappa(\eta j(\tau)+I_1(\tau))\frac{\sin^2 \varkappa_1 z_1}{\varkappa_1^2}} = 1, \quad (16)$$

where $\varkappa_1 = eh_1d/2mc^2\beta\gamma$; here we neglect the effects of the magnetic field on the beam current j_x arising for $h_1 \neq 0$, as $V_x << V_z$ by virtue of condition (9). In the stationary case

$$N_d = \sqrt{\frac{1}{1-\varkappa(\eta+I_1)\frac{\sin^2\varkappa_1 z}{\varkappa_1^2}}}, \quad (17)$$

where I_1 is defined by (16):

$$\frac{I_1}{\varkappa_1} F\left(\varkappa_1,\ \varkappa(\eta+I_1)\frac{1}{\varkappa_1^2}\right) = 1, \quad (18)$$

and $F(\varphi, k^2)$ is an elliptic integral of the first kind.

Figure 4 shows K as a function of $\varkappa$ for various $\varkappa_1$ ($1 - \varkappa_1 = 0$, $2 - \varkappa_1 = \pi/2$, $3 - \varkappa_1 = 3\pi/2$); from (16) and (17) we can derive $\varkappa_*$ for the case of an external magnetic field. For $\varkappa_1 \geqslant \pi/2$ the total current I_1 in discharge focusing is zero and

$$\varkappa_* = \eta\varkappa_1^2,$$

and for

$$\varkappa_1 < \frac{\pi}{2} \quad I_1 = \frac{\varkappa_1}{\sin \varkappa_1 F(\pi/2, \sin \varkappa_1)}$$
$$\text{and } \varkappa_* = \frac{1}{\eta + I_1}\frac{\varkappa_1^2}{\sin^2\varkappa_1}. \quad (19)$$

For $\varkappa_1 << 1$, (19) naturally coincides with (15), and an increase in $\varkappa_1$ naturally corresponds to an increase in $\varkappa_*$.

As we have pointed out above, our results are correct for a fairly narrow range in the beam and discharge parameters, and therefore they are qualitative rather than quantitative. For example, the approximations adequately represent high-power discharges with relatively small transverse dimensions induced by high-current beams of high-energy electrons. Under such conditions, the electron movement in the beam is influenced mainly by the magnetic field of the spatial discharge, whose contribution to the spatial distribution of the electrical energy in the gas has been incorporated. As criteria for the applicability of this model we have $\lambda \gg d$, $U_b \gg eU_0/mc^2$, where λ is the beam-electron scattering lengths for the gas molecules.

The case of a thin discharge is the most interesting, i.e., a discharge with small l/d, where (9) is obeyed even for high inhomogeneity in the energy deposition. For example, condition (9) in the stationary case with $\varkappa = \varkappa_*$ can be written as $(1/2)(l/d)^2 \ll 1$.

These results show that an externally maintained discharge with $\varkappa$ of the order of $\varkappa_*$ will be substantially inhomogeneous in space, and therefore the creation of a high-power laser based on a compressed gas requires a more detailed study of the physical processes responsible for homogeneity in absorption of the electrical energy in the spatial discharge.

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